Regression evidence and Kapitza's demographic projections

Discussion note

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1 Problem and data

Sergey Kapitza in his book [K] claims that the world population has been and continues to be growing at a more than exponential pace. This is a challanging hypothesis since it is unplausible that even exponential growth can be sustained over a long period of time by any natural or technical process. It is the purpose of this note to question the hypothesis and, ideally, trigger its discussion.

Unfortunately, the evidence for so-called hyperbolic population growth provided in [K] is thin to say the least. The approach is motivated by some vague analogy to electro magnetism, kinetic theory of gases and to the information society. But, after a hyperbolic type of growth function has been derived, it is not compared to any competing approach. This is a strong bias in favour of something "new" that may or may not be valid.

The hyperbolic hypothesis is derived from the ordinary differential equation

$$\frac{dN}{dx} = \frac{N^2}{K^2}$$

with some given constant $K > 0$, see [K, equation (3.2) on p. 71 and equation (A15) on p. 231]. The function N of time x continuously approximates the world population. The pivotal claim expressed by this ODE is that population increase is proportional to the square of the population size. This claim contrasts with the hypothesis that population increase is proportional to the mere population size. The latter results in the ODE

$$\frac{dN}{dx} = \frac{N}{K^2}.$$

This linear ODE has the exponential solution

$$N(x) = c \cdot e^{x/K^2}$$

with some unknown constant $c > 0$. Kapitza's ODE is of the Bernoulli type and has the solution

$$N_{K,d}(x) = \frac{K^2}{d\,K^2 - x}$$

with some unknown constant $d > 0$. These functions are hyperbolas with pole or so-called blow-up time $d\,K^2$. The last ODE is now generalized in order to interpolate between the exponential and the hyperbolic solutions. The ODE becomes

$$\frac{dN}{dx} = \frac{N^\alpha}{K^2}$$

with given constants $K > 0$ and $\alpha > 1$. The value $\alpha = 2$ denotes exactly the Kapitza case. The solution manifold of this ODE consists of the all hyperbolic functons

$$N(x) = \Big(\frac{\frac{K^2}{\alpha-1}}{d\cdot\frac{K^2}{\alpha-1} - x}\Big)^{\frac{1}{\alpha-1}} \text{ or } N(x) = \Big(\frac{\frac{b}{d}}{b-x}\Big)^{\frac{b}{K^2\cdot d}}$$

with unknown constant $d > 0$. The poles or blow-up times of the hyperbolic functions are $b = d\cdot\frac{K^2}{\alpha-1}$. Since all solution functions are hyperbolic, the exponential solution for $\alpha = 1$ is not covered directly but results from the limiting process $\alpha \to 1+$. The blow-up times then diverge to infinity.

The original Kapitza hypothesis is now addressed by the question whether the exponential functions or the hyperbolic functions have the better least squares fit to population data. It were strong supportive evidence for the Kapitza hypothesis if the parameter α at value two or close to value two led to the best among all hyperbolic fits and if, also, this were clearly better than the best exponential fit.

The empirical data for all regressions are given in the subsequent table.

Index i	Year x_i	Population y_i
1	1830	1 bn
2	1930	2 bn
3	1960	3 bn
4	1975	4 bn
5	1988	5 bn
6	2000	6 bn
7	2013	7 bn
8	2028	8 bn

World population data from [K, p. 28] in billions; 1 bn = 10^9.

2 Exponential regression

Least squares regression for the two-parametric exponential functions $c\cdot exp(x/K^2)$ and for the Kapitza population data is the minimization problem

$$\min_{a>0,\,K>0} \sum_{i=1}^{8} \big(c\cdot exp(x_i/K^2) - y_i\big)^2.$$

This best fit problem is solved by the parameters $K = 8.66$ and $c = 0.0149$ with regression error $5.885\cdot 10^{17}$ or standard deviation (root of regression error) $767.1\cdot 10^6$. The original data and the least squares regression function are plotted in figure 1; all computations and graphics done with Scilab [Sci].

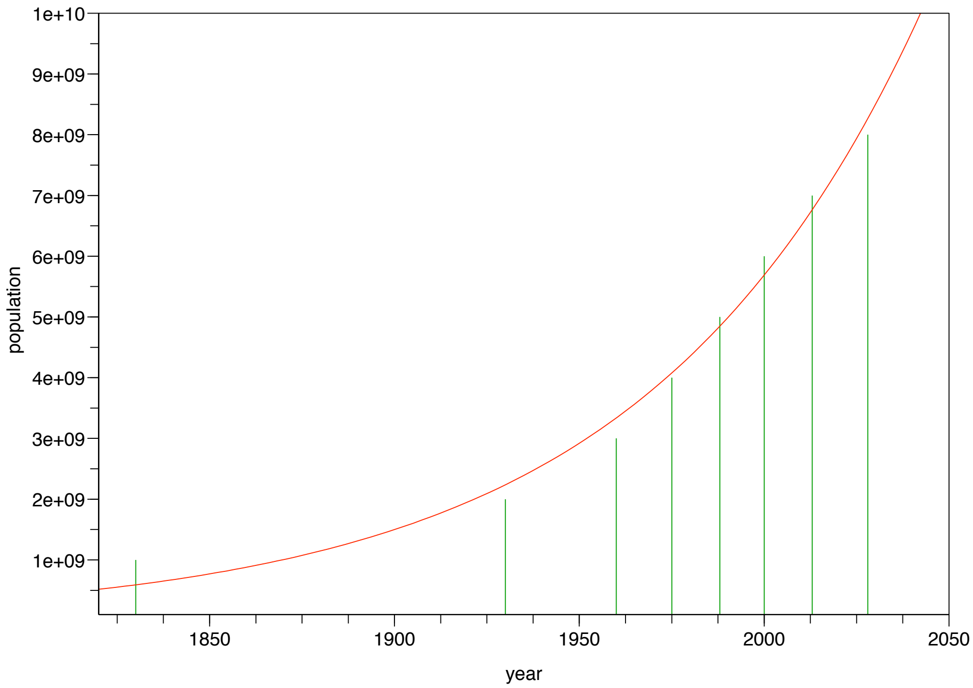

Figure 1: Population data and best fit exponential regression function.

3 Hyperbolic regression

Least squares regression for the three-parametric hyperbolic functions $(\frac{b/d}{b-x})^{\frac{b}{K^2 \cdot d}}$ for population blow-up at b and for the foregoing population data is the minimization problem

$$\min_{d>0,\, K>1, b>2050} \sum_{i=1}^{8} \left((\frac{b/d}{b-x_i})^{\frac{b}{K^2 \cdot d}} - y_i \right)^2 .$$

This best fit is attained for the parameters $b = 2383$, $K = 76$ and $d = 0.080$ with regression error $6.618 \cdot 10^{17}$ or standard deviation $813.5 \cdot 10^6$. The situation is depicted in figure 2. In particular, the regression error is slightly worse than for exponential regression and the best fit blow-up time is computed as year 2383. The first, fifth, sixth and seventh data point lie above and the other four data points lie below the best fit curve both in the exponential and the hyperbolic case. Though somewhat difficult so see, the best fit hyperbolic curve is slightly stronger curved than the best fit exponential curve.

4 Conclusion

Data driven regression seem to slightly favour the exponential approach over the hyperbolic approach. First, the exponential fitting error is slightly lower than the hyperbolic fitting error. Second, the parameters of the best hyperbolic fit correspond to $\alpha = 1.194$ which is closer to the exponential case $\alpha \to 1+$ than to the pure hyperbola which is given by $\alpha = 2$.

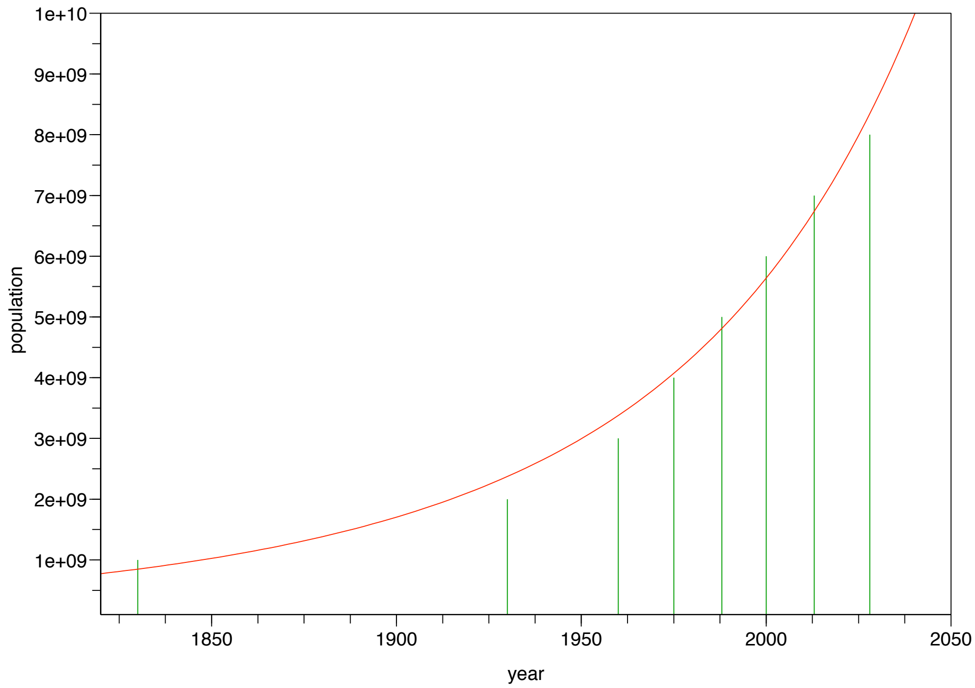

Figure 2: Population data and best fit hyperbolic regression function.

No evidence could be found that the world population grows faster than exponentially as claimed in [K]. It must be beared in mind that all empirical data more or less fluctuate around their true law – provided that it exists – and that the exponential law "explains" the given data as good as all hyperbolic competitors.

References

[K] Kapitza, S.P., "Global population blow-up and after", Global Marshall Plan Initiative, Hamburg, 2006.

[Sci] Scilab, Computer algebra system, version 4.1.2 with X11 graphic system, version 1.1.3 under MAC OS 10.4.10, source: www.scilab.org resp. www.hds.utc.fr/∼mottelet/Darwin.